Faculty of Science, Technology, Engineering and Mathematics
MU123 Discovering mathematics

MU123

EMA 2019B

Covers the whole module **Cut-off date 18 September 2019**

Submission instructions

There are some important differences in the procedure for preparing and submitting this End-of-module Assessment (EMA) compared to that for submitting TMAs.

Before beginning this assignment, you must read the EMA page on the MU123 website for all instructions relating to the EMA.

Your score out of 5 for good mathematical communication (GMC) will be recorded against Question 7. You do not have to submit any work for Question 7.

WEB 06686 3
19.1

Question 1 – 2 marks

Think back over your experience of studying MU123. Think about what has gone well, or what has not gone so well.

(a) Describe one change that you have made, or plan to make, to your studying. [*1*]

(b) Describe how you hope that the change will make your studying more effective. (If you think that no changes are necessary, then explain why.) [*1*]

Question 2 – 18 marks

The Department for Education is interested in researching local variations in the cost of school trips. Two sets of 20 teachers were asked what they think is a fair price (per pupil) for a school trip. The first set of teachers is based in London and the second set is based in Manchester. The data are recorded in Table 1.

Table 1 Responses of teachers to question:
'What is a fair price (per pupil) for a school trip?'

Teachers in London (£ per pupil)	Teachers in Manchester (£ per pupil)
0	0
5	0
5	0
7	3
8	3
10	3
10	4
10	4
15	4
15	4
18	6
20	6
20	5
20	5
20	5
20	10
20	10
25	10
25	12
28	12

(a) (i) Enter the responses into two new lists in Dataplotter. Copy and complete the table below. Round values, where necessary, to one decimal place. (The mean, rounded to one decimal place, is given for you as a check that you have entered the data correctly.) [*2*]

	Teachers in London (£ per pupil)	Teachers in Manchester (£ per pupil)
Min		
Median		
Max		
Mean	15.1	5.3
SD		
IQ range		
Range		

(ii) Is this an example of continuous or discrete data? [*1*]

(iii) Use the two measures of location to say which group has responded with a greater price on average. [*2*]

(iv) Use all three measures of spread to say which group has the greater variability in their responses to the question. [*2*]

(b) (i) Create boxplots for these two datasets (either drawn by hand or as a printout from Dataplotter). Include all the relevant information required for drawing boxplots as set out in Subsection 1.2 of Unit 11.

The summary values can be displayed on the boxplots themselves or in a table to the side of the chart, as they appear in Dataplotter. [*3*]

(ii) Use the boxplot for the responses from the teachers in London to say whether the data are symmetric or skewed. If the data are skewed, then state whether they are skewed to the left or skewed to the right. What does this tell you about how the data are spread in the dataset for teachers in London? [*2*]

(iii) The researcher from the Department for Education tries to summarise what the boxplots are telling her. Are the following statements true or false? In each case justify your answer.

(1) About three-quarters of the teachers in Manchester thought that the price per pupil should be £8 or less. [*2*]

(2) There were more teachers in London saying that the price per pupil should be more than £16.50 than teachers in London saying that the price per pupil should be less than £16.50. [*2*]

(c) The researcher also asks about the travel time that the two groups would find acceptable. A summary of the data is recorded in Table 2.

Table 2 Maximum travel time for a school trip

	Teachers in London (hours)	Teachers in Manchester (hours)
Min	0	0.1
Median	8.5	1.5
Max	21	2.5
Mean	6.9	1.3
SD	5.4	0.8
IQ range	9.7	1.4
Range	21	2.4

The researcher creates histograms for the travel time data, shown in Figure 1, but forgets to label them. Which histogram represents the Manchester teachers? Explain your answer. [2]

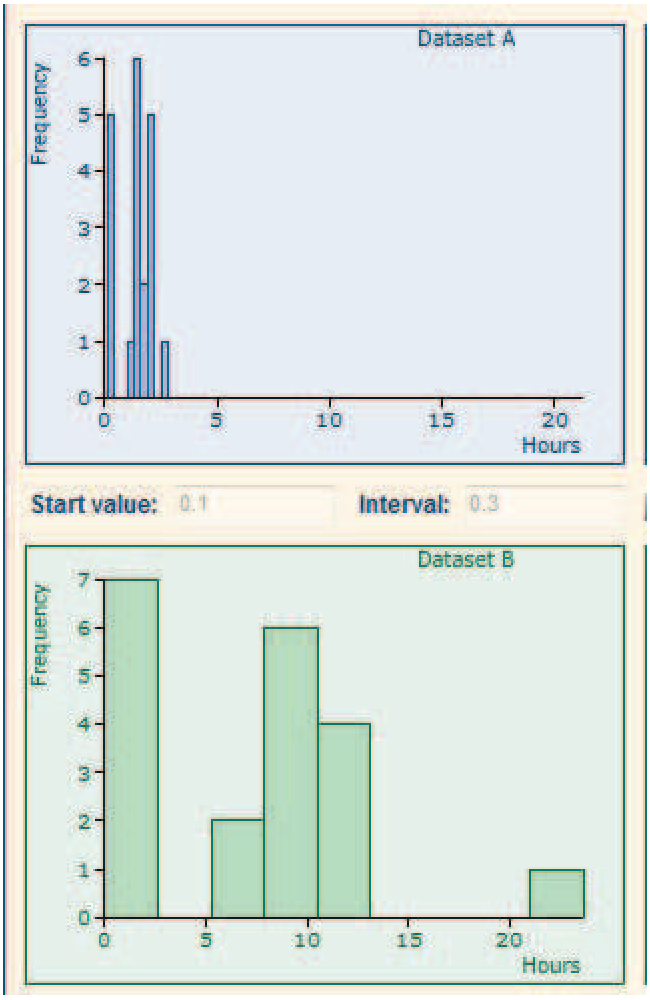

Figure 1

Question 3 – 19 marks

Throughout this question, use algebra to work out your answers. You may use a graph to check that your answers are correct, but it is not sufficient to read your results from a graph.

(a) A straight line passes through the points $(-12, 14)$ and $(3, -1)$.

(i) Calculate the gradient of the line. [*1*]

(ii) Find the equation of the line. [*2*]

(iii) Find the x-intercept of the line. [*2*]

(b) Does the point $(2, -2)$ lie on the line that you found in part (a)(ii)? Explain your answer. [*2*]

(c) Find the coordinates of the point where the lines with the following equations intersect:

$$-6x + y = 2,$$
$$9x + 5y = 10.$$

[*3*]

(d) A computer programmer is creating a mobile game called 'Irate Avians'. The game involves launching a missile from the left side of the screen to try to hit a target on the other side of the screen. The programmer is testing the game, and fires a test shot. The trajectory of the missile can be modelled by the equation

$$y = -0.23x^2 + 1.87x + 1.5,$$

where x is the horizontal distance of the missile from the launching point, and y is the vertical distance of the missile above the horizontal line representing the ground (both measured in centimetres).

(i) Find the y-intercept of the parabola $y = -0.23x^2 + 1.87x + 1.5$ (that is, find the height at which the missile leaves the launching point). [*1*]

(ii) (1) By substituting $x = 3$ into the equation of the parabola, find the coordinates of the point where the line $x = 3$ meets the parabola. [*2*]

(2) An obstacle of height 4 cm stands in the path of the missile's trajectory at a point 3 cm horizontally from the launching point. Using your answer from part (d)(ii)(1), state whether the missile will be able to clear this obstacle when it is launched. [*1*]

(iii) (1) Find the x-intercepts of the parabola. Give your answers in decimal form, correct to two significant figures. [*4*]

(2) Assuming that the missile lands on the ground, write down the horizontal distance from the launching point to where the missile lands. [*1*]

Question 4 – 17 marks

You should use algebra in all parts of this question, showing your working clearly.

(a) Solve the following equations, giving your answers as integers, fractions or surds in their simplest form.

(i) $16x + 3 = -4x - 7$ [2]

(ii) $10 + \frac{1}{2}(4 - 2x) = \frac{1}{5}x + 6$ [3]

(iii) $\dfrac{x}{x-3} - \dfrac{1}{x-4} = 1$ [4]

(b) Solve the equation $2x^2 + 38x + 140 = 0$ by factorisation. [3]

(c) A student was asked to rearrange the formula

$$-\frac{1}{3}a + 6b = 2b - 3c\left(\frac{3b}{2} - 5\right)$$

to make b the subject. The student's **incorrect** attempt is shown below.

The equation is: $-\frac{1}{3}a + 6b = 2b - 3c\left(\frac{3b}{2} - 5\right)$

Multiply both sides by 3: $-a + 18b = 6b - 9c\left(\frac{3b}{2} - 5\right)$

Multiply out brackets: $-a + 18b = 6b - \frac{27bc}{2} - 45c$

Multiply both sides by 2: $-2a + 36b = 12b - 27bc - 90c$

Collect b terms on one side: $24b - 27bc = 2a - 90c$

Factorise left-hand side: $b(24 - 27c) = 2a - 90c$

Divide both sides by $24 - 27c$ to give: $b = \dfrac{2a - 90c}{24 - 27c}$

(i) Write out a correct rearrangement of the formula. [3]

(ii) Identify and explain, as if directly to the student, two of the three mistakes made by the student. [2]

Question 5 – 20 marks

Throughout this question, take care to explain your reasoning carefully. You should round your answers, where necessary, to two significant figures.

Satellites that orbit the Earth at a distance of 35 800 km above the Earth's equator, and travel at a speed that matches the speed of the Earth's rotation, are said to be in geostationary orbit. In this orbit, a satellite's distance above the Earth's surface is fixed, and it maintains a fixed position above a certain point on the Earth's surface as it rotates.

A telecommunications company owns satellites A, B and C, which are in geostationary orbit. Satellite A is diametrically opposite satellite C (i.e. AC is the diameter of the geostationary orbit). Satellite B is 51 000 km from satellite A, and $\angle ABC$ is 90°. Assume that the Earth is spherical, with diameter 12 700 km. A diagram of the satellites in orbit is given in Figure 2.

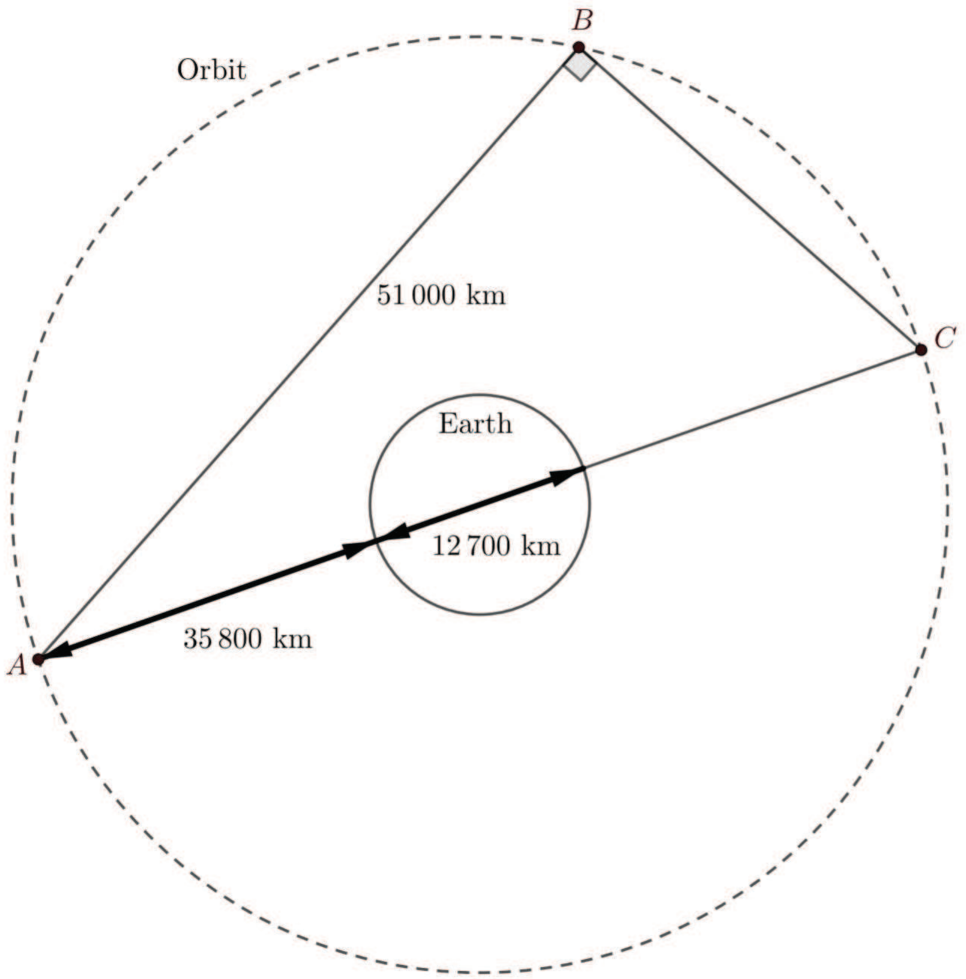

Figure 2

(a) (i) Find the length AC. [*1*]

(ii) Find the circumference of the geostationary orbit. [*2*]

(iii) Use Pythagoras' Theorem to calculate the length BC. [*2*]

(iv) Using trigonometry, calculate $\angle BAC$. [*3*]

(b) A relay station D is built on Earth between satellites A and B. $\angle ADB$ is 145°, and $\angle DAB$ is 16°. This information is shown in Figure 3.

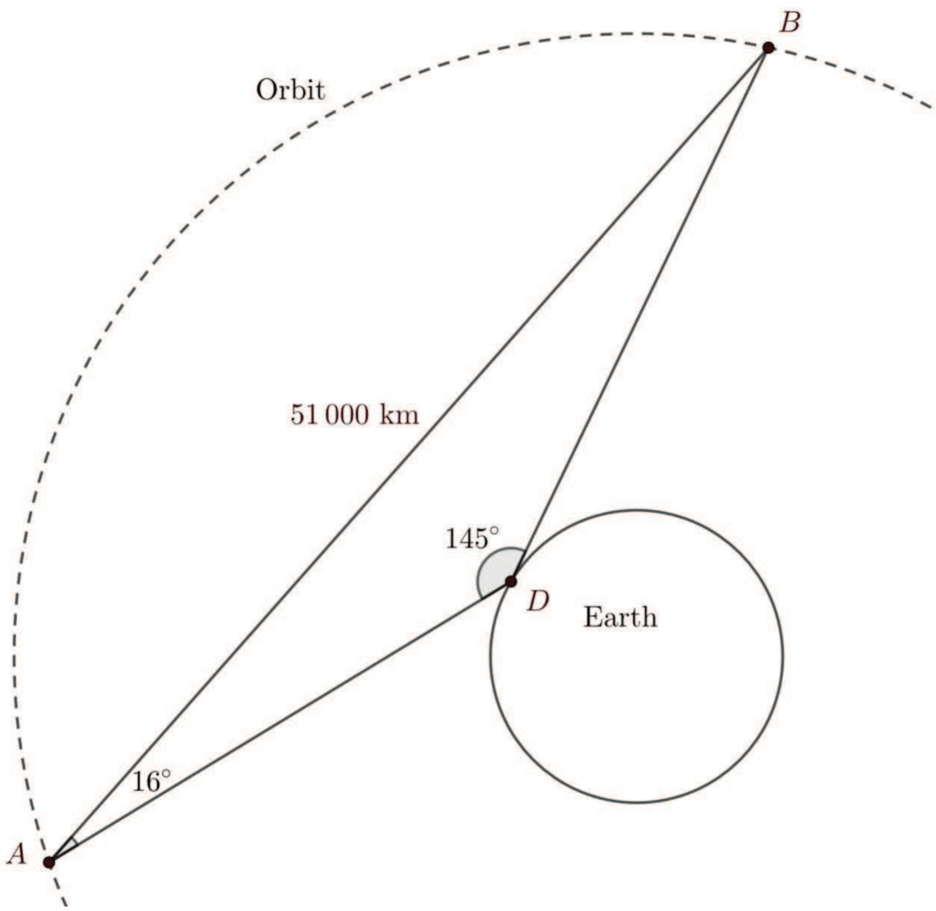

Figure 3

(i) Is triangle ABC similar to triangle ADB? Explain your answer. [*1*]

(ii) Use the Sine Rule to calculate the length BD. [*4*]

(iii) Find the area of triangle ADB. [*3*]

(c) The radius of the Moon is 1700 km, and the radius of the Earth is 6350 km. A scale model of the Earth and the Moon is made. In the model, the radius of the Moon is 3.4 cm.

(i) Find the radius of the scale model of the Earth. [*2*]

(ii) Find the volume of the scale model of the Earth. [*2*]

Question 6 – 19 marks

In this question, where necessary, you should round answers appropriately.

(a) A dog owner records the weight of her dog. She finds that from the age of 20 weeks to the age of 48 weeks, the dog's weight can be modelled by the equation

$$w = 0.92t - 0.15 \quad (20 \leq t \leq 48),$$

where w is the weight of the dog in kilograms, and t is the age in weeks.

(i) Find the weight of the dog at age 26 weeks according to this model. [*2*]

(ii) Explain the inequality $(20 \leq t \leq 48)$ that follows the equation. [*1*]

(iii) Using algebra, calculate the age at which the dog's weight is 40 kg. [*3*]

(iv) Write down the gradient of the straight line represented by the equation

$$w = 0.92t - 0.15.$$

What does this measure in the practical situation being modelled? [*2*]

(v) Either using Graphplotter or by hand, sketch the graph of

$$w = 0.92t - 0.15,$$

putting w on the vertical axis and covering the time interval $20 \leq t \leq 48$. [*1*]

(vi) Explain why the model cannot be extended to model accurately the dog's weight at birth. [*2*]

(b) The same dog owner observes that from birth to age 14 weeks, the puppy's weight can be modelled by the equation

$$w = 365 \times (1.23)^t \quad (0 \leq t \leq 14),$$

where w is the weight of the puppy in grams, and t is the age in weeks.

(i) Calculate the weight of the puppy at age 10 weeks. [*2*]

(ii) Write down the scale factor, and use this to find the percentage increase in the weight each week. [*2*]

(iii) Use the method shown in Unit 13, Subsection 5.2 to find the age at which the puppy will weigh 4000 grams. [*4*]

Question 7 – 5 marks

A score out of 5 marks for good mathematical communication over the entire EMA will be recorded under Question 7.